AN EXPERIMENTAL DETERMINATION OF THE TRANSITION TO TURBULENCE USING THE ELECTRIC DOUBLE LAYER

By
ROLAND CARL ANDERSON

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LIST OF SYMBOLS

A	cross-sectional area
c	wave speed
d	diameter of a tube
e	electronic charge
E	streaming potential
i_1	electrical convection current
i_2	electrical conduction current
k	Boltzmann constant
ℓ	length of a tube
n_i	number density of i^{th} ion species
$\vec{n}$	unit normal vector
N	coordinate normal to a wall
P	pressure
r	coordinate measured from the center of a tube
R	mean radius of a tube
R_n	Reynolds number
s	arc length
t	time
T	temperature
u	velocity in Poiseuille flow
$\bar{u}$	mean velocity
$\vec{v}$	velocity vector
x	Cartesian coordinate
y	Cartesian coordinate
z_i	valence of the i^{th} ion species

α	wave number
γ	charge density
δ	distance of the Stern layer from a wall
ϵ	dielectric constant
ϕ	wave amplitude function
λ	specific conductance
μ	absolute viscosity
ν	kinematic viscosity
ρ	fluid density
ψ	potential in the electric double layer
ψ'	perturbation of the stream function
τ_w	fluid shear stress at the wall
χ	reciprocal of Debye length

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Roland Carl Anderson

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Major Department: Aerospace Engineering

The experimental determination of the critical Reynolds number for transition from laminar to turbulent flow in a straight circular pipe has been accomplished by use of a streaming potential technique. The streaming potential was generated by an electrolyte (0.001 M KCl) flowing through a capillary tube with a diameter of 0.0242 inches. It was found that, at the onset of turbulence, the streaming potential began to fluctuate in an easily identifiable manner. This fluctuation, combined with mass flow measurements, was used to determine the critical Reynolds number to be 1907 ± 3 for a fixed geometry and fluid.

I INTRODUCTION

1. Scope

The purpose of the present investigation is twofold. First, a new technique using the "electric double layer" is developed which is capable of measuring hydrodynamic effects within Ångstroms of a wall. Secondly, this method is applied to the old, but as yet theoretically unresolved, problem of the transition from laminar to turbulent flow in a straight pipe of circular cross-section.

2. Stability of Poiseuille Flow

The theoretical problem of determining the stability of Poiseuille flow has been the subject of investigations for many years; considerable efforts have been made for both the plane and circular cases. Interesting results have been obtained in both cases although the former case has been by far the most fruitful.

Plane Poiseuille flow has been studied on the basis of two dimensional infinitesimal disturbances where the perturbation in the stream function is of the form

$$\psi'(x\,y\,t) = \phi(y)\,e^{i\alpha(x-ct)}$$

where ψ' is the perturbation in the stream function,

ϕ is the amplitude,

α is the wave number,

c is the speed of propagation,

and t is time.

The rectangular Cartesian coordinates (x,y) are taken parallel and perpendicular to the flow. This form is not quite as restrictive as it might seem since Squire(1) has shown that the three dimensional case is equivalent to the two dimensional one at a lower Reynolds number. Heisenberg(2) was the first to deduce the instability of plane flow for sufficiently large Reynolds number, but he did not calculate a critical value beyond which instability would occur. The method of Heisenberg was extended by Tollmien(3) and Schlichting(4) and was used to calculate the stability curves for the boundary layer on a flat plate. Later, Lin(5) clarified the previous work and succeeded in calculating the neutral curve for plane Poiseuille flow. Contrary to the conclusion of Lin, Pekeris[6] applied a different technique and concluded that Poiseuille flow was stable for all Reynolds numbers. To resolve this disagreement, Thomas[7] calculated the critical Reynolds number by direct numerical methods and found it to be 5780. However, one should bear in mind that the above work is for infinitesimal two dimensional disturbances and that nothing has been successfully worked out for an arbitrary perturbation.

The results for pipe flow are not as rewarding as the plane case. Sexl[8] was the first to solve the viscous stability equations for axisymmetric disturbances. However, his results are questioned because of the mathematical simplifications used. Pretsch[9] found that if the analysis is restricted to a thin layer near the wall, the problem becomes the same as for a disturbance near the wall in Couette flow. Pekeris[10] found a different solution for a region near the center of the pipe. Corcos and Sellars[11] have obtained a solution which accounts for the work of both Pretsch and Pekeris, however, their solution is in terms of a finite set of eigenfunctions. Schensted[12] has resolved this question and has shown that for axisymmetric disturbances the set of eigenfunctions are not only infinite but also complete. All the existing investigations of the parabolic profile tend to show that the flow is stable with respect to infinitesimal two dimensional disturbances. Of course, it is well known that turbulent flow does exist in an experimental sense in a circular pipe. Two possible explanations of this paradox have been proposed, one by Meksyn and Stuart[13] and the second by Tatsumi[14]. Meksyn and Stuart showed that in a channel the minimum critical Reynolds number decreased as the amplitude of the oscillations increased, thus suggesting that instability in a circular pipe is the result of finite oscillations. On the other hand, Tatsumi considered the stability in the inlet section of the pipe where the flow is more of the boundary layer type, he calculated a minimum critical Reynolds number of the order of 10^4, whereas the maximum Reynolds number which has been obtained experimentally by Taylor (unpublished) and Ekman[15] for laminar flow is 4×10^4.

3. Experimental Investigations of Transition in a Pipe

The experimental study of the stability of flow through pipes of circular cross-section has occupied a prominent position in the history of fluid mechanics since the time of Osborne Reynolds. In 1883, Reynolds(16) reported the results of experiments which showed beyond doubt that there exist two possible modes of flow in the same pipe--laminar and turbulent. In addition to the experiments, Reynolds reasoned dimensionally that the flow in pipes of different diameters should be similar if the dimensionless number (Reynolds number) $\frac{\rho \bar{u} d}{\mu}$ is the same in each pipe where ρ is the density of the fluid, d is the diameter, a characteristic linear dimension, $\bar{u}$ is the mean velocity and μ is the absolute viscosity.

Reynolds performed two independent sets of experiments. In the first group, which is the most widely known, he introduced dye into the mouth of a tube while the velocity was increased. He found that at low velocities the dye would go through the tube as a filament, but, as the velocity increased, a point would be reached where the filament would very suddenly spread out and fill the downstream end of the tube. This point he called the critical velocity. The results of these measurements are best summarized by Reynolds:

> "These experiments had shown definitely that there was a critical value of the velocity at which eddies began if the water were approximately steady when drawn into the tube, but they

> had also shown definitely that at such critical velocity the water in the tube was in a highly unstable condition, any considerable disturbance in the water causing the break down to occur at velocities much below the highest that could be attained when the water was at its steadiest, suggesting that if there were a critical velocity at which, for any disturbance whatever, the water became stable, this velocity was much less than that at which it would become unstable for infinitely small disturbances; or, in other words, suggesting that there were two critical values for the velocity in the tube, the one at which steady motion changed into eddies, the other at which eddies changed into steady motion."

Reynolds proceeded to determine this lower critical velocity. The experiment was to introduce initially turbulent water "from the Manchester main" into a long straight lead pipe and to measure the pressure loss over a 5 foot length on the downstream end The critical velocity was taken as the point where the pressure gradient was no longer proportional to the velocity. His results showed, in modern terminology, a lower critical Reynolds number of approximately 2000

Since 1883, many workers have repeated Reynolds' experiments, the most extensive repetition of Reynolds' work was conducted by Stanton and Pannell(17). In addition, several interesting variations of the classical method have been devised. For example, Barnes and Coker(18) used a thermal scheme where the walls of the tube were heated and the onset of turbulence was detected by noting a sharp rise in the temperature in the interior of the flow. They found the critical Reynolds number equal to 1900. Also, in 1962, Reiss and Hanratty(19)(20) developed a technique from which they could infer the behavior of the flow within the so-called "laminar sublayer" by measuring the mass transfer to a small sink on the wall. They used a polarized electrode, which was current limited by mass transfer, as a sink. Thus, any fluctuation in the concentration would result in fluctuations at the wall. Using this scheme, they found random fluctuations within the sublayer and from the onset of these fluctuations deduced the critical Reynolds number to be 2100.

4 The Electric Double Layer

It is well known from the study of colloids that at a solid-liquid interface, the solid surface becomes charged. The charges on the wall are supplied by the liquid but just how they are attached or even where they are attached is still open to question. Therefore, following Helmholtz[21], the experimental evidence that such a charged layer exists shall be accepted in the present work. Since there is a charge residing on the interface, there must also be an equal and opposite layer of charge in the adjacent region. These charged layers are called the electric double layer.

The early workers[21][22] in the field considered the double layer to be composed of two distinct layers of charge, one fixed to the wall and the other free to move with the fluid. Gouy[23] and Chapman[24] independently formulated the theory of the diffuse double layer where the charge in the fluid is a function of the distance from the wall. This is in essence the theory of ionic atmospheres given some ten years later by Debye and Hückel[25]. Gouy and Chapman considered the double layer to be ions attached to the solid phase and an equivalent ionic charge of opposite sign distributed in the solution near the interface.

The charge on the wall is treated as a uniform surface charge. The space charge in the solution is due to the unequal distribution of ions which are assumed to have no physical size. The solvent is considered as a continuous medium entering the problem only through its dielectric constant. Coulomb interactions in the system are described by Poisson's equation

$$\nabla^2 \psi = -\frac{4\pi\gamma}{\epsilon} \tag{1}$$

where ψ is the potential (having a value of ψ_0 at the wall),

γ is the charge density,

ϵ is the dielectric constant,

and ∇^2 is the Laplacian operator.

The number density of the i^{th} ion species is assumed to be given by

$$n_i = n_{i0} \exp(-z_i e \psi / kT) \tag{2}$$

where n_i is the number density of the i^{th} species,

n_{i0} is the number density of the i^{th} species at the wall,

z_i is the valence,

e is the electronic charge,

k is Boltzmann's constant,

and T is the temperature of the solution

The charge density is given by

$$\rho = \sum_{i} z_i e n_i \tag{3}$$

Combining equations (1), (2), and (3), one obtains

$$\nabla^2 \psi = -\frac{4\pi}{\epsilon} \sum_{i} z_i e n_{i0} \exp\left(-z_i e \psi / kT\right) \tag{4}$$

which is the differential equation for the potential as a function of the space coordinates. If the assumption of an infinite flat wall is made, equation (4) reduces to

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi}{\epsilon} \sum_{i} z_i e n_{i0} \exp\left(-z_i e \psi / kT\right) \tag{5}$$

which is subject to the boundary conditions

$$\psi = \frac{d\psi}{dx} = 0 \qquad \text{for } x \to \infty$$

where x is the distance from the wall

Letting $\chi^2 = \frac{4\pi e^2 \sum_i n_{io} z_i^2}{\epsilon k T}$,

the solution of (5) for a single species is

$$\chi x = \ln \frac{[\exp(ze\Psi/2kT)+1][\exp(ze\Psi_o/kT)-1]}{[\exp(ze\Psi/2kT)-1][\exp(ze\Psi_o/kT)+1]} \tag{6}$$

This can be simplified, if $ze\Psi/kT$ is assumed small, by expanding the exponentials. The result is

$$\chi x = \ln \Psi_o/\Psi$$

or

$$\Psi = \Psi_o e^{-\chi x} \tag{7}$$

showing that Ψ drops by a factor of e^{-1} in the distance $1/\chi$. This value is called the thickness of the double layer or Debye length. As an example, this distance is approximately 10^{-6}cm for a 0.001 M, KCl solution. This theory was found to be an over simplification since the concentration of the electrolyte is calculated to be 300M at the wall for a 0.1 M solution and a wall potential of 200 mv. This is an impossible value.

The Gouy-Chapman theory was modified by Stern[26] to allow for the finite size of the ions and thus to avoid the difficulty mentioned above. Stern suggested that, since the influence of the dimensions of the ions is greatest at the wall, it is perhaps possible to add a major correction to double layer theory by accounting for the finite size of the ions only in the region of the first few molecular layers. This simply requires that the Gouy-Chapman theory be applied not at the wall but a short distance, δ , away from it. In addition, Stern considered the possibility of adsorption of the ions in the plane $x = \delta$ This layer has become known as the Stern layer.

5. Streaming Potential

The streaming potential is caused by the flow of an electrolyte sweeping out ions in the movable part of the double layer. This flow causes an electrical current by the convection of charges parallel to the wall which, in turn, results in a potential difference between the two ends of the passage[27] This potential difference causes a conduction current to flow which at equilibrium is equal and opposite to the convection current.

Consider a tube of arbitrary cross-section with coordinates as shown in Fig. 1. Assuming the charge density on the wall is independent of the flow velocity, the convection current, i_1 , one finds that

$$i_1 = \int_A \rho \vec{v} \, \vec{n} \, dA \tag{8}$$

where ρ is the charge density,

$\vec{v}$ is the hydrodynamic velocity,

$\vec{n}$ is the unit normal to dA,

A is the cross-sectional area,

dA is equal to dNds,

N is the coordinate normal to the wall,

and S is the arc length.

If Poisson's equation is used, equation (8) becomes

$$i_1 = -\frac{\epsilon}{4\pi} \int_A \nabla^2 \psi \, \vec{v} \, \vec{n} \, dA \tag{9}$$

It is now assumed that the thickness of the double layer is small compared to the dimensions of the tube. In this case $\nabla^2 \psi$ becomes $\frac{d^2\psi}{dN^2}$ and (9) may be written as

$$i_1 = -\frac{\epsilon}{4\pi} \int_A \frac{d^2\psi}{dN^2} v_n \, dA \tag{10}$$

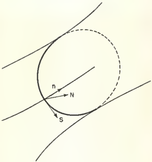


Fig 1 Coordinate system for tube of arbitrary cross-section

Integration by parts and the application of the boundary conditions

$$\Psi = \frac{d\Psi}{dN} = 0 \qquad \text{as} \qquad N \to \infty$$

and

$$\vec{V} = 0 \qquad \text{at} \qquad N = 0$$

yields

$$I_1 = -\frac{\epsilon}{4\pi}\iint \frac{dv_n}{dN}\frac{d\Psi}{dN}\, dN\, ds$$

$$= -\frac{\epsilon}{4\pi}\oint \Psi_0 \frac{dv_n}{dN}\, ds - \frac{\epsilon}{4\pi}\iint \Psi \frac{d^2 v_n}{dN^2}\, dN\, ds \tag{11}$$

where $\Psi_0 = \Psi(N)\big|_{N=0}$

If the flow is laminar,

$$\frac{dv_n}{dN}\Big|_{N=0} = \frac{\tau_w}{\mu}$$

where μ is the absolute viscosity,

and τ_w is the shear stress at the wall.

Also, if the flow is in a constant area channel,

$$\oint \frac{\tau_w}{\mu}\, ds = -\iint \frac{1}{\mu}\frac{dP}{dn}\, dN\, ds.$$

Therefore equation (11) becomes

$$i_1 = \frac{\epsilon\psi_0}{4\pi\mu}\iint \frac{dP}{dn}\,dN\,ds - \frac{\epsilon}{4\pi}\iint \psi\,\frac{d^2 v_n}{dN^2}\,dN\,ds \tag{12}$$

The second term in equation (12) will be neglected since it may be shown that the ratio of the second term to the first is*

$$O\left(\frac{1/\kappa}{R}\right)$$

where R is the mean radius of the tube. Hence, if the pressure gradient is assumed constant over the cross-section, equation (12) may be written as

$$i_1 = \frac{\epsilon\psi_0}{4\pi\mu}\frac{dP}{dn}A \tag{13}$$

The conduction current, i_2 , is

$$i_2 = -\iint_A \lambda \frac{dE}{dn}\,dA \tag{14}$$

where λ is the specific conductance

and E is the potential caused by the fluid flow.

---

*The notation $O(x)$ means "of order χ "

If $\frac{dE}{dn}$ and λ are constant over a cross-sectional area, (14) becomes

$$i_2 = -\lambda \frac{dE}{dn} A \tag{15}$$

Equilibrium requires $i_1 = -i_2$, hence

$$\frac{dE}{dP} = \frac{\epsilon \psi_0}{4\pi\mu\lambda} \tag{16}$$

Under the above assumptions, equation (16) shows that the streaming potential is a linear function of the pressure drop along the tube. The first term on the RHS of equation (11) indicates that the convection current, and hence the streaming potential, should be sensitive to the velocity gradient at the wall. Thus a change in the streaming potential should provide a means of detecting a change in $\frac{dv_x}{dy}$.

II THE EXPERIMENT

1. General Description

The theory given in section 5 of Chapter I shows that the streaming potential varies linearly with the pressure gradient along the passage. Also, equation (11) shows that the streaming potential varies linearly with the velocity gradient at the wall. It is this relation that has produced a new technique for hydrodynamic measurements.

Let us now restrict the discussion to the flow in a straight tube with a circular cross-section. If the flow is laminar, the velocity distribution becomes

$$u = \frac{R^2}{4\mu}\frac{dp}{dl}\left(1 - \frac{r^2}{R^2}\right)$$

at a sufficient distance from the inlet, where

R is the radius of tube,

p is the pressure,

l is the distance along the tube,

r is the distance from the center of the tube,

and u is the velocity

If this relation is substituted into equation (11), the streaming potential is again found to vary linearly with the pressure gradient along the tube. This has been observed by physical chemists for many years(28). If the flow changes from laminar to turbulent, the velocity gradient at the wall no longer varies linearly with the pressure gradient, and one therefore expects a change from a straight line to a nonlinear curve when plotting streaming potential versus pressure drop along the tube. In addition, the streaming potential might be expected to fluctuate because of the turbulent eddies causing changes in velocity near the wall. As a result of these predicted effects at the onset of turbulence, an experiment was constructed to attempt to determine the lower critical Reynolds number using streaming potential measurements.

2 Laboratory

The laboratory was especially designed and constructed for the present experiments. This was necessary since the work is particularly sensitive to temperature and humidity. The strong temperature dependence is due to the viscosity and density of the fluid and even though extremely precise temperature control was achieved, the changes in viscosity are still the overriding errors. The humidity of the room comes into play in the electrical measurements. The currents involved are of the order of 10^{-8} to 10^{-8} amps with a source impedance of the order of 50 meg ohms. Hence, surface leakage could be a problem.

The laboratory is a 10 x 15 foot room inside an air-conditioned building. The entire room is vapor sealed with 10 mil polyethylene. The walls and floor are insulated with 3 inches and the ceiling with 4 inches of Styrofoam. The air-conditioning controls are of the potentiometer type which control a servo-driven back pressure regulator installed in the compressor suction line. This provides control of the room temperature to ± 0.75°F over the range 40-70°F. The ± 0.75°F is a continuous variation with approximately a 4 minute period. Since the quantity of fluid in the experiment is 5 liters of water, the thermal inertia is large resulting in only small drifts in temperature--0(0.1°C)--over a period of an hour.

The air drying is accomplished by use of a compression-cooling-expansion cycle. This enables the room to be held below 20% R.H. during operation.

3. Apparatus

The apparatus used is schematically shown in Fig. 2. The reservoirs R_1 and R_2 are 5 liter boiling flasks fitted with ground glass taper joints, J_1 and J_2. Tubes L_1 and L_2 are 8 mm Pyrex with taper joints at the top for inserting electrodes. A_1 and A_2 are 8 mm tubes terminating with the female portion of ground glass ball joints. The tube T is a precision bore capillary tube 0.0242 ± .0003 inches in diameter, 4.8 inches long. The ends are flared into 8 mm tubes which end in the male portion of the ball joints. In operation, the liquid is forced up L_1, over a glass trip at B (see Fig. 3) and through T by applying gas pressure through V_1. The levels in the flasks are determined by connecting either S_1 or S_2 to a height gage (see Fig. 4). The level can be determined within ± 0.002 inch.

The electrodes, E_1 and E_2, were Ag-AgCl constructed on a spiral of 24 AWG platinum. The Ag-AgCl coating was made with a mixture of 90% silver oxide and 10% silver chlorate formed into a paste and painted on the platinum wire. The electrode was then baked at 500°C for fifteen minutes.

Fig. 2 Schematic sketch of experimental apparatus.

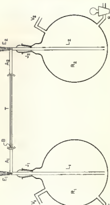

V2
E2
S2
J2
A2
L2
R2
T
B
A1
J1
L1
R1
E1
V1
S1

Fig. 3 Photograph of the glass trip used in the islet of the precision bore tube

Fig. 4 Height gage manometer used for determining the water level in the reservoir

The gas pressure was supplied by commercially bottled nitrogen. A sketch of the pressure system is shown in Fig. 5. The gas was reduced in pressure by a standard pressure regulator obtained from the local gas company. It was found in the beginning that such a regulator could not control the pressure to the desired accuracy so the needle valve and the bleed valve were installed. The bleed valve was designed for this application and is shown in Fig. 6. This valve consists simply of a 1 mm capillary tube with provisions for a 26 gage wire to slide in and out of the bore. The pressure drop through the tube depends on the position of the wire. This setup worked very well, providing a means of controlling the pressure on the flasks very precisely. The manometer was a simple mercury manometer used in combination with a microscope and a height gage (see Fig. 7). With this optical setup, a change in pressure of 0.001 inch of mercury could be detected. This combined with the precision bleed valve allowed the pressure to be controlled easily to within ± 0.005 inch Hg. The two three-way stopcocks in the supply line provided a convenient means of either applying pressure to the flasks or venting them.

Fig. 5 A sketch of the pressure system.

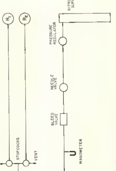
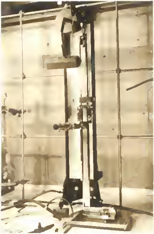

R1
R2
NITROGEN SUPPLY
PRESSURE REGULATOR
NEEDLE VALVE
BLEED VALVE
MANOMETER
VENT
STOPCOCKS
VENT

Fig. 6 Bleed valve.

Fig 7 Manometer and optical system used
for pressure measurement



4. Cleaning

One of the more difficult problems in the experiment was the cleaning of the experimental components. This included cleaning the glassware and electrodes and maintaining rather rigid purity controls All this was desirable since contaminants could cause inconsistent results

The following procedure was used for the glassware. The glassware was soaked in chromic acid for two hours and then rinsed in 160°F conductivity water. While the glass was still hot, chromic acid was added again and allowed to stand for another hour. It was then rinsed four times in hot conductivity water and steamed in the same quality water for four hours. If the surface of the glass did not wet uniformly when steaming began, it was treated again with alternate doses of chromic acid and hot water until the wetting was uniform. Steaming was then continued for four hours.

The electrodes were cleaned in a 10% solution of nitric acid for two hours and leached in conductivity water for 12 hours.

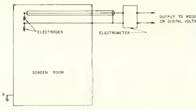
The conductivity water was obtained from a commercially built still. The still was adjusted so the condensate came out at approximately 190°F. This was collected in a 9 liter Pyrex serum bottle with a ground glass stopper. Before being used, the bottle was cleaned as described above. During collection, nitrogen was bubbled through the condensate. This helped to remove any remaining carbon dioxide and maintained a nitrogen atmosphere in the bottle, preventing reabsorption of carbon dioxide and ammonia from the air. Any water with a specific resistance lower than 4.0 x 10^6 ohm-cm was rejected.

The solution used in the experiments was 0.001 M KCl, which resulted in a double layer thickness of approximately 10^{-6} cm. This was made using conductivity water and reagent grade KCl which had been recrystallized in the same quality water. The mixing of this solution was timed so that it could be placed in the flasks as soon as the steaming process had ended. The entire apparatus was then sealed immediately. The sealing was accomplished using red sealing wax; the wax was never allowed to come into contact with the solution. The flasks were never reopened until the completion of the experiments. The procedure seemed to be successful in that the streaming potentials were quite consistent over a period of about two months.

5 Electrical Measurements

The electrical measurements were afflicted with the usual problems found in electrochemistry, i.e., low currents, $0(10^{-9}$ amps), and high source impedance, $0(50$ meg ohms) Thus the measuring device selected had very low current drain and the entire experiment was shielded from stray electrical noise. A schematic diagram for the measurements is shown in Fig 8. The experiment was enclosed in a screen room which was grounded at G by a 20 foot ground rod. A General Radio Model number 1230-A Electrometer, having an input impedance of 10^{11} ohms, was used as a preamplifier. The electrodes were connected to the electrometer by a coaxial conductor passing through the wall of the screen room The output of the electrometer was read in two ways depending on the experiment. If the average value of the streaming potential was desired, the output was connected directly to a Dymec 2401A integrating digital voltmeter. If the fluctuations of the streaming potential were to be studied, the output voltage of the electrometer was biased by a d.c. voltage so the resulting signal could be recorded by a 10 inch Varian strip chart recorder operating on a full scale range of 5 mv This arrangement proved to be quite satisfactory The results are shown in Chapter III.

Fig. 8 Electrical diagram for measurement and shielding.


OUTPUT TO RECORDER
OR DIGITAL VOLTMETER
ELECTRODES
ELECTROMETER
SCREEN ROOM
G

6. Experimental Procedures

The final procedure used to determine the critical Reynolds number was to measure the pressure where a particular type of fluctuation (to be discussed in Chapter III) in the streaming potential was found. This pressure was then used to find the corresponding Reynolds number from calibration curves obtained from mass flow measurements. The calibration curve is shown in Fig. 9. This curve is a linear least squares fit for the indicated data points.

Fig 9   Tube calibration curve

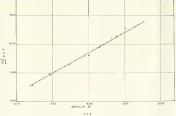


FIG. 8

III. RESULTS

1. Measurements

The first measurements taken were the average streaming potential as a function of the pressure differential causing the flow. A typical plot of $\frac{Pd^3}{\mu\nu\ell}$ versus E is shown in Fig. 10 where

P is the pressure,

d is the diameter of the tube,

μ is the absolute viscosity,

ν is the kinematic viscosity,

ℓ is the length of the tube,

and E is the streaming potential.

It is seen that there seems to be a slight break in the curve in the region $\frac{Pd^3}{\mu\nu\ell} = 1030$. This is emphasized by the two straight lines drawn through the data points. However, the break is so slight, it is difficult to select accurately the pressure where this occurs. To avoid this problem, the data were taken at pressure intervals of 0.1 inches of Hg (instead of 0.5 inches as used in Fig. 10) and then differentiated one time. In

Fig. 10 Typical plot of $\frac{Pd^3}{\mu v \ell}$ vs E.

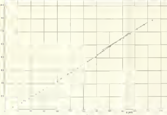
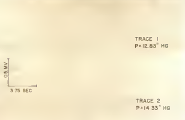


Fig. 13

this way, it was thought the break could be detected with a great deal more precision. The method was unsuccessful. It turned out that in the region of interest the sample period of the integrating voltmeter was too short (1 sec.) to give a steady reading to the desired accuracy. The readings would oscillate roughly ± 0.1 mv. Since the change in streaming potential was of the order of 10 mv/ inch Hg, this represented an error of ± 10% in the measured slope of the curve for pressure increments of 0.1 inch. The error completely obliterated the change in slope at the critical Reynolds number.

A decision was made to use this troublesome noise rather than try to eliminate its effects. To do this, it was necessary to look at a small noise component which had a characteristic peak-to-peak amplitude of 0.25 mv and which was additionally superimposed on a d.c. voltage of the order of 125 mv. This was accomplished, as indicated in Chapter II, by biasing the signal with an opposing d.c. voltage so that the noise could be recorded on a 10 inch strip chart recorder with a 5 mv full scale deflection. Typical records are shown in Figs. 11, 12, 13 and 14 for a temperature of 18.76°C. Traces 1 and 2 of Fig. 11 show

Fig 11 Typical traces for low flow rates


TRACE 1
P=12.83" HG
0.5 MV
3.75 SEC
TRACE 2
P=14.33" HG

the noise level for pressures of 12.83 and 14.33 inches of Hg. No change in the traces can be seen from zero flow up to a region just below transition. Traces 3 and 4 of Fig. 12 show a gradual increase in noise up to transition.

First thoughts were that the beginning of this buildup represented instability in the flow. However, the fact that it increased gradually could not be explained in the light of the results of earlier experimenters who observed the onset of turbulence to be quite rapid and violent. Thus, the search was continued at higher and higher pressures to see if there was anything to indicate a sudden change in affairs within the flow. Indeed, such an indication was found. This is shown by traces 5 and 6 in Fig. 13. The sharp spikes rising well above the background appeared very suddenly and became more numerous as the pressure was increased. A very curious thing to be noted is that spikes always represented an increase in voltage when they first appeared. As the pressure is increased to a sufficiently high value, the spikes become positive and negative and completely dominate the trace. Such a trace is shown in Fig. 14.

Fig. 12 Typical traces showing the buildup of noise as transition is approached.

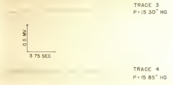

TRACE 3
P = 15.30" HG
0.5 MV
3.75 SEC
TRACE 4
P = 15.85" HG

Fig 13 Typical traces showing the background noise and the spikes caused by instability in the flow

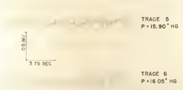
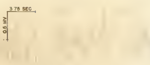

TRACE 5
P = 15.90" HG
0.5 MV
3.75 SEC
TRACE 6
P = 16.05" HG

Fig 14 Typical trace for flow well above the critical state.



TRACE 7
P = 17.50" HG

Having completed this study, it was reasoned that the initial noise, shown in traces 3 and 4, was caused by disturbances from the trip at the mouth of the tube which had not been completely damped before reaching the wall. The velocity at which the high peaks first began was taken as the critical velocity. Using this criterion, 32 measurements were made at various temperatures to determine the critical Reynolds number. The results are tabulated in Table 1. The pressure is included in the table in its dimensional form to indicate the sensitivity of the results to temperature variations. Notice that a temperature change from 20.54°C to 18.50°C required a change in pressure from 15.68 to 16.63 inches of Hg in order to reach transition. The mean value of the Reynolds number was found to be 1907 with a mean error of ± 3.27. The standard deviation of the measurements with respect to the mean is 19.5. This value of 1907 ± 3 is in excellent agreement with 1900 found by Barnes and Coker (loc. cit.) and with the recent work of Lessen, Fox, Bhat and Liu(29) where a range from 1400 to 2000 was reported.

Table 1

Tabulated Results of the Experiment

P	T	R_e crit
16 66	19 54	1649
16 66	19 54	1649
15 66	20 54	1617
15 63	20 54	1611
16 78	19 21	1628
16 76	19 21	1626
16 23	19 70	1666
16 43	19 66	1628
16 63	19 60	1626
15 73	20 46	1614
15 73	20 46	1614
15 73	20 48	1614
16 46	19 62	1626
16 46	19 58	1621
15 48	20 78	1612
15 48	20 78	1612
15 43	20 67	1615
15 50	20 48	1690
15 45	20 46	1694
15 50	20 48	1690
15 45	20 48	1664
15 55	20 46	1697
15 50	20 40	1660
15 40	20 40	1683
15 45	20 42	1664
15 50	20 42	1660
15 45	20 42	1683
16 05	16 70	1680
16 10	16.70	1686
15 60	16 84	1676
16 75	15 6	1683
16 80	15 6	1687

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Roland C. Anderson was born in Trenton, N. J., on November 27, 1934. He attended the public schools of Fort McCoy, Florida, and was graduated from Fort McCoy High School in 1952. In September, 1952, he began his college education at the University of Florida and, during this first year, participated in inter-collegiate athletics as a member of the Florida Freshman Basketball Team.

At the end of the freshman year his education was interrupted by four years of service with the United States Air Force Security Service.

In September, 1957, formal education was resumed at the University of Florida where, in June, 1960, he received the degree of Bachelor of Aeronautical Engineering with High Honors. He remained at the University of Florida for graduate work. The degree of Master of Science in Engineering, with a major in Aerospace Engineering and minors in Mathematics and Physics, was earned in August, 1961.

Mr. Anderson became a member of the faculty of the University of Florida in September, 1961, and began to work, part time, for the Ph.D. degree. In June, 1962, he became a member of the faculty at West Virginia University.

Again, graduate work was continued and, in addition, flight training was taken at the West Virginia University Flight School.

Mr. Anderson then returned to the University of Florida staff in January, 1963, where he has remained until the present.

Mr. Anderson is married and has two children.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy

August, 1965

R. Martin
Dean, College of Engineering

Dean, Graduate School

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